Recovery of Interblock information

In all incomplete block designs there are two types of blocks. One type of blocks consists of sets of k treatment numbers. Another type of blocks are the actual blocks, each consisting of a set of k experimental units. The two types of blocks are treatment blocks and unit blocks.

The randomization procedure in incomplete block designs consists of first allotting the treatments blocks to the unit blocks at random. Then the treatments thus allotted to a unit block are distributed at random to the different units in the unit blocks. As the treatment sets are allotted to the blocks at random, the block effects can be considered to be random variables.

Analysis with Recovery of inter block information:

Block effects can be considered to be random variables in incomplete block designs.

Accordingly, using block effects as another source of error, there can be more estimate of the treatment effects. Such estimates are called inter-block estimates.

Rao (1947) gave a method of obtaining combined estimates of treatments effect for incomplete block designs in general.

The model is

$$y_{ij} = \mu + t_i + b_j + e_{ij}$$

e_{ij}'s alone are considered to be random variables with a constant variance for intra block analysis. For interblock analysis b_j's are also considered as random variables and hence the random variables y_{ij} have the variance

$\frac{1}{k}(k\sigma_b^2 + \sigma^2)$, where σ_b^2 is the block variance.

The inter block estimates are obtained by treating the block totals as observations, and are obtained by minimizing.

$$\sum_j \left(B_{j\cdot} - k\mu - \sum_i n_{ij} t_i \right)^2 \rightarrow ①$$

w.r.t μ and t_i.

The normal equations are

$$G = bk\bar{\mu} + r \sum_i \bar{t}_i \rightarrow ②$$

and

$$V_i = kr\bar{\mu} + r\bar{t}_i \sum_{i' \neq 1} \left(\sum_j n_{ij} n_{i'j} \right) \bar{t}_{i'}$$

$; (i = 1, 2, \ldots, v)$

$\rightarrow ③$

where V_i is the sum of the totals of those blocks in which the i^{th} treatment occurs and G is the grand total.

From ② and ③, we get

$$r\bar{t}_i + \sum_{i' \neq 1} \lambda_{ii'} t_{i'} = V_i - \frac{r}{b} G = P_i, \; (i = 1, 2, \ldots, v)$$

$\rightarrow ④$

where

$$\lambda_{ii'} = \sum n_{ij} n_{i'j} \quad P_i = V_i - \frac{r}{b} G.$$

The reduced normal equations for intra block estimates which are obtained by minimizing

$$\sum_{ij} (y_{ij} - t_i - b_j)^2 \rightarrow (5)$$

are $r(k-1)\bar{t}_i - \sum_{i' \neq i} \lambda_{ii'} \bar{t}_{i'} = k Q_i \quad (i=1,2,\ldots v)$

$$\rightarrow (6)$$

Let $W = \frac{1}{\sigma^2}$ and $\frac{W'}{k} = \frac{1}{k} = (k\sigma_b^2 + \sigma^2)^{-1}$.

Then linearly combining the two equations (4) and (6) with W'/k and W respectively as weights we get the following equation from which the combined estimates can be obtained:

$$\frac{r}{k}\{W(k-1) + W'\}\bar{t}_i - \frac{W-W'}{k}\sum_{i' \neq i} \lambda_{ii'} \bar{t}_i$$

$$= W Q_i + \frac{W'}{k} P_i \rightarrow (7)$$

The equation (7) can also be ~~ob~~ obtained by first combining the two least squares set-ups

1 and 5 with W'/k and W respectively as weights and then minimizing w.r.t the parameters.

The equation 7 are the general reduced normal equations for all incomplete block designs.